## Punched cards



01000000000000000000000000000111111110100001000101000010000100001000000000000000



 444441444444441444444441444444144444444441111444444444444444444444444444444444420| $555555 \mathbf{I} 5555555$ I55555555I5555555I5555555555555555II5555555I55551555555555555555 6666666I66666666I66666666I6666666I66666666666666I66666I6I6666I6666666666666666666
 8888888881888888881888888881888888818IIII8IIIB8IIIIBIIIIUIItII 88888888888888 9999999999 І9999999999999999919999999199999999999999999999999999999999999999999999


## Some history

In the first half of the 20th Century, IBM's flagship product was the Tabulator, which is pretty much just a big adding machine. Beginning about 1930, IBM began to produce machines that could also multiply, and eventually divide, as well as add and subtract. These were called Calculators and they were used primarily for engineering and scientific applications.

## IBM 601 Calculator

First IBM (electro-mechanical) calculator that could multiply.


931

## The idea of computing



- $A, B, C, D$ - data: "stream" (file/deck) of punched cards
- $A+B, C+D$ - partial result: (file/deck) of punched cards
- $(A+B)(C+D)$ - result: the stream of punched cards


## IBM 601 Calculator, cont.

The IBM 601 Multiplying Punch read two factors up to eight decimal digits in length from a card and punched their product onto a blank field of the same card. It could subtract and add as well as multiply. It had no printing capacity, so was generally used as an offline assistant for a tabulator or accounting machine. The 601 that was delivered to Eckert's lab in 1933 was a special model "capable of doing the direct interpolation, a very unusual feature, specially designed for Eckert by one of IBM's top engineers at Endicott [NY]". Eckert went a step further by connecting the 601 to a Type 285 Tabulator and a Type 016 Duplicating Punch through a calculation control switch of his own design, forming the first machine to perform complex scientific computations automatically.
http://www.columbia.edu/cu/computinghistory/601.html

## Early digital (electro-mechanical) computers. . .

...follow this idea, as well.


## Project Manhattan

Richard P. Feynman used to administer the group of "human computers" used for performing computations for the Manhattan Project. He assisted in establishing a system for using IBM punched cards for computation.
You can find this story on-line, described by Feynman.
This was described in the book Surely You're Joking, Mr. Feynman! by Richard Feynman and Ralph Leighton.

## John von Neumann

1. One of the most important computational projects in the forties (20th century) were calculations for the atomic bomb.
2. John von Neumann, a brilliant mathematician, and physicist (who was also involved in quantum mechanics, game theory, computer science, functional analysis ...) were engaged in this work.

## von Neumann architecture

So-called "von Neumann architecture" (presented in First Draft of a Report on the EDVAC) describes the architecture for a computer with subdivision of a processing unit, a control unit, a memory, external mass storage, and input and output mechanisms.

- processing unit has an arithmetic logic unit and processor registers (kind of a local storage),
- memory was used to store both data and instructions,
- control unit has an instruction register and program counter,
von Neumann architecture



## von Neumann architecture

Computer system build according to von Neuman's architecture allows for:

- inputting a program from an external source to the computer's memory,
- inputting data and changing them easily.
- In general, the data and the program residing in teh computer's memory are indistinguishable.
- Processor or strictly speaking, computer program can modify itself.
- The information is processed by sequential execution of program instructions.

Wrodaw University of Science and Technolo

## von Neumann architecture

- These conditions allow for switching from the execution of one task (program) to another without physical intervention in the structure of the system and thus ensure its versatility.
- Von Neumann's computer system does not have separate memory for storing data and instructions. Instructions and data are encoded in the form of numbers. Without an analysis of the program is difficult to determine whether the area of memory contains data or instructions.
- The executed program can modify itself treating its own instructions (code) as data: changing them and then executing.


## Simplified diagram of a computer



## Simplified diagram of a computer



## Simplified diagram of a computer



## Simplified diagram of a computer



## Simplified diagram of a computer



## Simplified diagram of a computer



## Simplified diagram of a computer

In this model:

- Processor.
- Memory (all kinds of it: RAM, ROM, Disks, floppies, external disks, and so on).
- All input and output devices allowing for communication with "external word": keyboard, mouse, touch-pad, printer, graphic card. . .
- Control: all electronic devices allowing for fetching and executing program's instructions from the memory, and transferring instructions and data through the bus.
- Bus: all connections allowing for a flow of data and control instructions.


## Calculator

| Display |  |  |  |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |

## Calculator

| Display |  |  |  |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | C | $=$ | $\div$ |

- It is the simplest calculator: four operations only.


## Calculator

| Display |  |  |  |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | C | $=$ | $\div$ |

- It is the simplest calculator: four operations only.
- Only keyboard (digits, operations) and display.


## Calculator

| Display |  |  |  |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | C | $=$ | $\div$ |

- It is the simplest calculator: four operations only.
- Only keyboard (digits, operations) and display.
- After pressing digit keys (123) data is copied to the display.


## Calculator

| 1 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | + |  |
| 4 | 5 | 6 | - |  |
| 7 | 8 | 9 | $*$ |  |
| 0 | $C$ | $=$ | $\div$ |  |

- It is the simplest calculator: four operations only.
- Only keyboard (digits, operations) and display.
- After pressing digit keys (123) data is copied to the display.


## Calculator

| 12 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | + |  |
| 4 | 5 | 6 | - |  |
| 7 | 8 | 9 | $*$ |  |
| 0 | $C$ | $=$ | $\div$ |  |

- It is the simplest calculator: four operations only.
- Only keyboard (digits, operations) and display.
- After pressing digit keys (123) data is copied to the display.


## Calculator

| 123 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | + |  |
| 4 | 5 | 6 | - |  |
| 7 | 8 | 9 | $*$ |  |
| 0 | $C$ | $=$ | $\div$ |  |

- It is the simplest calculator: four operations only.
- Only keyboard (digits, operations) and display.
- After pressing digit keys (123) data is copied to the display.


## Calculator

| 123 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | + |  |
| 4 | 5 | 6 | - |  |
| 7 | 8 | 9 | $*$ |  |
| 0 | $C$ | $=$ | $\div$ |  |

- It is the simplest calculator: four operations only.
- Only keyboard (digits, operations) and display.
- After pressing digit keys (123) data is copied to the display.
- There should be some kind of a memory for storing a keyed number. This memory on one side is connected to the keyboard (data input) on the other to the display (data output).


## Calculator

| 123 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | + |  |
| 4 | 5 | 6 | - |  |
| 7 | 8 | 9 | $*$ |  |
| 0 | $C$ | $=$ | $\div$ |  |

- It is the simplest calculator: four operations only.
- Only keyboard (digits, operations) and display.
- After pressing digit keys (123) data is copied to the display.
- There should be some kind of a memory for storing a keyed number. This memory on one side is connected to the keyboard (data input) on the other to the display (data output).
- Let's modify our diagram.


## Calculator

| Display |  |  |  |
| :---: | :---: | :---: | :---: |
| Accumulator |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | C | $=$ | $\div$ |

- It is a simplest calculator: four operations only: keyboard (digits, operations) and display

Wrodaw University of Science and Technolo

## Calculator

| Display |  |  |  |
| :---: | :---: | :---: | :---: |
| Accumulator |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | C | $=$ | $\div$ |

- It is a simplest calculator: four operations only: keyboard (digits, operations) and display
- You can not see memory called accumulator, but probably it exists.


## Calculator

| Display |  |  |  |
| :---: | :---: | :---: | :---: |
| Accumulator |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | C | $=$ | $\div$ |

- It is a simplest calculator: four operations only: keyboard (digits, operations) and display
- You can not see memory called accumulator, but probably it exists.
- After pressing digit keys (123) data is stored in the memory and displayed on the display.


## Calculator

|  |  |  |  |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |

- It is a simplest calculator: four operations only: keyboard (digits, operations) and display
- You can not see memory called accumulator, but probably it exists.
- After pressing digit keys (123) data is stored in the memory and displayed on the display.


## Calculator

| 12 |  |  |  |
| :---: | :---: | :---: | :---: |
| 12 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |

- It is a simplest calculator: four operations only: keyboard (digits, operations) and display
- You can not see memory called accumulator, but probably it exists.
- After pressing digit keys (123) data is stored in the memory and displayed on the display.


## Calculator

| 123 |  |  |  |
| :---: | :---: | :---: | :---: |
| 123 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |

- It is a simplest calculator: four operations only: keyboard (digits, operations) and display
- You can not see memory called accumulator, but probably it exists.
- After pressing digit keys (123) data is stored in the memory and displayed on the display.


## Calculator

| 123 |  |  |  |
| :---: | :---: | :---: | :---: |
| 123 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |

- It is a simplest calculator: four operations only: keyboard (digits, operations) and display
- You can not see memory called accumulator, but probably it exists.
- After pressing digit keys (123) data is stored in the memory and displayed on the display.
- Let's press operation key (+)


## Calculator

| 123 |  |  |  |
| :---: | :---: | :---: | :---: |
| 123 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |

- It is a simplest calculator: four operations only: keyboard (digits, operations) and display
- You can not see memory called accumulator, but probably it exists.
- After pressing digit keys (123) data is stored in the memory and displayed on the display.
- Let's press operation key (+)


## Calculator

| 123 |  |  |  |
| :---: | :---: | :---: | :---: |
| 123 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |

- The display has not changed but pressing the digit key erases the display and the new value appears.


## Calculator

| 123 |  |  |  |
| :---: | :---: | :---: | :---: |
| 123 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |

- The display has not changed but pressing the digit key erases the display and the new value appears.
- The first value does not disappear; there must be another memory.


## Calculator

| 123 |  |  |  |
| :---: | :---: | :---: | :---: |
| 123 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |

- The display has not changed but pressing the digit key erases the display and the new value appears.
- The first value does not disappear; there must be another memory.
- All arithmetic operations have two arguments.


## Calculator

| 123 |  |  |  |
| :---: | :---: | :---: | :---: |
| 123 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |

- The first value does not disappear; there must be another memory.
- All arithmetic operations have two arguments.
- Additional memory stores the second argument.


## Calculator

| Display |  |  |  |
| :---: | :---: | :---: | :---: |
| Accumulator |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |
| Memory |  |  |  |

Now, probably all key elements are shown.

Wroclaw University of Science and Technolo

## Calculator

| Display |  |  |  |
| :---: | :---: | :---: | :---: |
| Accumulator |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |
| Memory |  |  |  |

## Operations

Accumulator

## Calculator

|  |  |  |  |
| :---: | :---: | :---: | :---: |
| 123 |  |  |  |
| 123 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |
| Memory |  |  |  |

## Operations

123

## Calculator

| 123 |  |  |  |
| :---: | :---: | :---: | :---: |
| 123 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |
|  |  |  |  |
| 123 |  |  |  |

Operations
123
$+$

## Calculator

|  |  |  |  |
| :---: | :---: | :---: | :---: |
| 55 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |
|  |  |  |  |
| 123 |  |  |  |

## Operations <br> 123 <br> $+$ <br> 55

## Calculator

| 178 |  |  |  |
| :---: | :---: | :---: | :---: |
| 178 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | C | $=$ | $\div$ |
|  |  |  |  |
|  |  |  |  |

Operations
123
$+$
55
$+$

## Calculator

| 22 |  |  |  |
| :---: | :---: | :---: | :---: |
| 22 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |
|  |  |  |  |
| 178 |  |  |  |

Operations
123
$+$
55
$+$
22

## Calculator

| 200 |  |  |  |
| :---: | :---: | :---: | :---: |
| 200 |  |  |  |
| 1 | 2 | 3 | + |
| 4 | 5 | 6 | - |
| 7 | 8 | 9 | $*$ |
| 0 | $C$ | $=$ | $\div$ |
|  |  |  |  |
|  |  |  | 178 |

Operations
123
+
55
+
22
$=$

Processor


## Basic operations

## Arithmetic

- Load <memory address> copies the data from a particular cell of a RAM into the accumulator.
- Store <memory address> copies the content of the accumulator to the memory.
- Load indirect <number $>$ copies the number into the accumulator.
- Add <memory address> adds content of memory cell to the current content of the accumulator. (We can assume that there are also operations for subtracting, multiplying, and dividing data; but this is not always true.)
Execution of each operation, changing the register (accumulator) sets the indicators (zero, overflow, negative).


## Basic operations

- Nagation Changes the sign of the number in the accumulator.
- And <memory address> logical AND (bitwise)
- Or <memory address>
- Xor <memory address> - Exclusive OR
- Shift_left
- Shift_right


## Control operations

- Jump <memory address> the next instruction will be taken from the given address
- Jump_if_zero <memory address>
- Jump_if_negative <memory address>
- Jump_if_overflow <memory address>
- Jump_to_subroutine <memory address> very similar to the normal jump instruction, but also saves the current state of the processor in a dedicated memory (allowing for restoration of the state later on).


## Assembler

## Very simple operation: <br> $A=B+C$

## Assembler

Very simple operation:
A=B+C
We have to put the result of the sum of values stored in the memory addresses B and C in the memory cell, having the address A.

Wrodaw University of Science and Technolo

## Assembler

Very simple operation:
$\mathrm{A}=\mathrm{B}+\mathrm{C}$
We have to put the result of the sum of values stored in the memory addresses B and C in the memory cell, having the address A.
Computer realisation:
Load B
Add C
Store A

## Assembler

More complicated example

$$
Z=\frac{[(A+B)(C+D)]}{W}
$$

## Assembler

More complicated example

$$
Z=\frac{[(A+B)(C+D)]}{W}
$$

$$
T 1=A+B
$$

## Assembler

More complicated example

$$
Z=\frac{[(A+B)(C+D)]}{W}
$$

$$
\begin{aligned}
& T 1=A+B \\
& T 2=C+D
\end{aligned}
$$

## Assembler

More complicated example

$$
Z=\frac{[(A+B)(C+D)]}{W}
$$

$$
\begin{aligned}
& T 1=A+B \\
& T 2=C+D \\
& T 3=T 1 * T 2
\end{aligned}
$$

## Assembler

More complicated example

$$
Z=\frac{[(A+B)(C+D)]}{W}
$$

$$
\begin{aligned}
& T 1=A+B \\
& T 2=C+D \\
& T 3=T 1 * T 2 \\
& Z=T 3 / W
\end{aligned}
$$

## MARIE

## MARIE - A Machine Architecture that is Really Intuitive and Easy

- base two positional system (2's complement)
- constant word size
- addressing of words
- 4Ki bytesof main memory (12 bits)
- 16-bit data (16-bit word)
- 16 -bit instruction ( 4 -bit operation code +12 -bit address)
- 16-bit accumulator (AC)
- 16-bit instruction register (IR)
- 16-bit memory buffer register (MBR)
- 12-bit program counter (PC)
- 12-bit memory address register (MAR)
- 8-bit Input register (InREG)
- 8-bit output register (OutREG)


Wrocław University of Science and Technolg

## MARIE Simulator





 \begin{tabular}{lllllllllllllllllll}
\hline OAO \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000

 $\begin{array}{llllllllllllllllllll}O B O & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000\end{array}$ 

\hline$O C 0$ \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000

 

\hline$O D 0$ \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 <br>
\hline $0 E O$ \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 <br>
\hline OFO

 

\hline$O F O$ \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000 \& 0000
\end{tabular} $\begin{array}{lllllllllllllllllllll}100 & 1105 & 3106 & 2104 & 7000 & 0000 & 0014 & 001 E & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000\end{array}$ $\begin{array}{llllllllllllllllllll}110 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & -\end{array}$

## Homework

## Simple program for MARIE computer simulator (for example multiplication of two

 numbers).Additional resources:


MARIE.
https://pl.wikipedia.org/wiki/MARIE.
Only in Polish.Student resources - essentials of computer organization and architecture, second edition.
http://samples.jbpub.com/9781284123036/9781284136852_FMxx_Print_Final.pdf.
Only Table of Contents and Preface.


Kuo-pao Yang.
MARIE: An introduction to a simple computer.
https://www2.southeastern.edu/Academics/Faculty/kyang/2013/Spring/CMPS375/ClassNotes/CMPS375ClassNotesChap04.pdf, 2013.
Linda Null and Julia Lobur.
The essentials of computer organization and architecture.
Jones and Bartlett Publishers, Sudbury, Mass., 2006.


Linda Null and Julia Lobur.
MARIE: an introduction to a simple computer.
In The essentials of computer organization and architecture. 2006.
http://samples.jbpub.com/9781449600068/00068_CH04_Null3e.pdf.


Linda Null and Julia Lobur.
A guide to the MARIE machine simulator environment, 2010.
https://cs.msutexas.edu/~simpson/wordpress/wp-content/uploads/2012/12/MarieGuide.pdf.
Wrodaw University of Science and Technolo:

## Reverse Polish Notation (RPN)

Let's see the operation

$$
3+7 \times 5
$$

What is the result?

## Reverse Polish Notation (RPN)

Let's see the operation

$$
3+7 \times 5
$$

What is the result?
50 or 38 ?

## Reverse Polish Notation (RPN)

Let's see the operation

$$
3+7 \times 5
$$

What is the result?
50 or 38 ?
Which one is correct?

## Reverse Polish Notation (RPN)

Let's see the operation

$$
3+7 \times 5
$$

What is the result?
50 or 38 ?
Which one is correct?
But, still, there is a lot of simple calculators that are "bad?"

## "Priority" of arithmetic operations

1. power
2. multiplying and dividing
3. summing and subtracting

We can use the parentheses for changing the order of operations.

## "Priority" of arithmetic operations

1. power
2. multiplying and dividing
3. summing and subtracting

We can use the parentheses for changing the order of operations.
BTW, What about changing the sign?

## Do there exist unambiguous notation?

Polish logician, Łukasiewicz, introduced "prefix notation". Instead of writing $z=x+y$ he proposed notation:

$$
+x y
$$

## Do there exist unambiguous notation?

Polish logician, Łukasiewicz, introduced "prefix notation". Instead of writing $z=x+y$ he proposed notation:

$$
+x y
$$

Let's note that it is very similar to writing a function of two variables:

$$
z=f(x, y)
$$

## Do there exist unambiguous notation?

Polish logician, Łukasiewicz, introduced "prefix notation". Instead of writing $z=x+y$ he proposed notation:

$$
+x y
$$

Let's note that it is very similar to writing a function of two variables:

$$
z=f(x, y)
$$

The sum function ( + ) has two arguments:

$$
z=+(x, y)
$$

## Polish Notation

Operation $3+7 \times 5$ mening $3+(7 \times 5)$ we can note:


## Reverse Polish Notation

For some practical reasons we use "postfix notation," writing the operation (operator) after its arguments:

$$
x y+
$$

We call this notation Reverse Polish Notation (RPN)
So we can write this operation like this:

$$
75 \times 3+
$$

and a more complicated example, as:

$$
A B+C D+\times W /
$$

## Reverse Polish Notation: Stack

Practical realization of this operation

$$
A B+C D+\times W /
$$

requires a stack and additional operations in internal language.

- Push writes accumulator's content to the stack.
- Pop takes value from the top of the stack and puts it in the accumulator.


## Reverse Polish Notation: Stack

Practical realization of this operation

$$
A B+C D+\times W /
$$

requires a stack and additional operations in internal language.

- Push writes accumulator's content to the stack.
- Pop takes value from the top of the stack and puts it in the accumulator.

The stack is, sometimes called LIFO (Last In First Out) memory. .

## Reverse Polish Notation: Stack

Practical realization of this operation

$$
A B+C D+\times W /
$$

requires a stack and additional operations in internal language.

- Push writes accumulator's content to the stack.
- Pop takes value from the top of the stack and puts it in the accumulator.

The stack is, sometimes called LIFO (Last In First Out) memory...
The queue (aka "line") is called FIFO (First IN First Out) memory...

The idea of a stack


